The Effect of Unified Desktop on the Waiting Time of Customers

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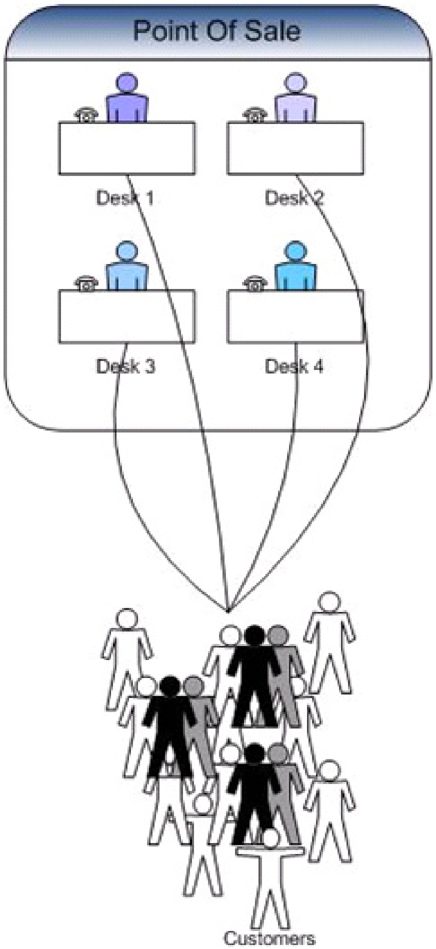
A **Unified Desktop** has a significant impact on the service quality. While the cost and effort of an optimization initiative can be estimated relatively easily in most cases, the quantification of the result in subject to the customers' waiting time is difficult.

The following article shows the calculation of the benefit in a **Point of Sale or shop**. It uses scientific insights in mathematics about stochastic processes, in particular the Queueing Theory. Interested readers can find references to further examination and mathematical derivation within the context.



THE SHOP

The Shop is a place with direct customer interaction. In the shop customers can order new contracts, buy phone accessories or can renew and change contracts. Four desks are installed in our model shop for processing customer requests. In average 20 customers enter the shop per hour. The existing tools allow a staff member to serve an average of six customers per hour. The shop works in a FCFS mode (First Come First Served).

The fact that customers don't come in constant intervals and the handling time varies heavily (instead of being exactly 10 minutes every time) complicates the calculation. Therefore the commonly used Poisson distribution is taken for the definition of the probability of the arrival times and the process times (M1).

The Kendall notation describes this environment as a M|M|4 system (M2).

THE OPTIMIZATION

We assume that the operator of the shop was able to improve the productivity of a customer representative by **10%** by using appropriate tools – which is a very conservative assumption. This means that now 6.6 customers can be served hourly on average.

This leads us to the following questions:

- **How does this improvement change the length of the waiting queue?**
- **How much is the average waiting time improved?**

EXAMINATION OF THE WAITING TIME AND THE QUEUE LENGTH

	Before optimization	**After sample optimization**
Arrival rate (M3)	$\lambda = 10 \frac{customers}{hour}$	$\lambda = 10 \frac{customers}{hour}$
Service time (M5)	$\overline{t_B} = 19 \frac{Minutes}{customer}$	$\overline{t_B} = 15 \frac{Minutes}{customer}$
Service rate (M4)	$\mu \approx 3{,}16 \frac{customers}{hour}$	$\mu \approx 4{,}00 \frac{customers}{hour}$
Traffic volume (M6)	$\rho = \frac{10}{4 \cdot 3{,}16} = 0{,}791$	$\rho = \frac{10}{4 \cdot 4{,}00} = 0{,}625$
Service rate of the system (M7)	$\mu_{System} = 12{,}64 \frac{customers}{hour}$	$\mu_{System} = 16{,}00 \frac{customers}{hour}$
Probability that no customer in the system (M8)	$P(X = 0) \approx 0{,}029$	$P(X = 0) \approx 0{,}073$
Probability that customer has to wait (M9)	$P(X \geq 4) \approx 0{,}582$	$P(X \geq m) \approx 0{,}320$
Average Queue length (M10)	$L_q \approx 2{,}2$	$L_q \approx 0{,}5$
Average waiting time (M11)	$W_q \approx 13 \text{min } 15 \text{sec}$	$W_q \approx 3 \text{min } 12 \text{sec}$

THE RESULT

Before optimization the average length of the waiting queue is 2.2 persons. In average every customer has to wait 13 minutes 15 seconds before getting served.

Changing the representative's productivity by 20% results in significant improvements – the average queue length and the average waiting time is reduced to a fourth.

In numbers: The average waiting time was reduced from about 13 Minutes to about 3 Minutes!

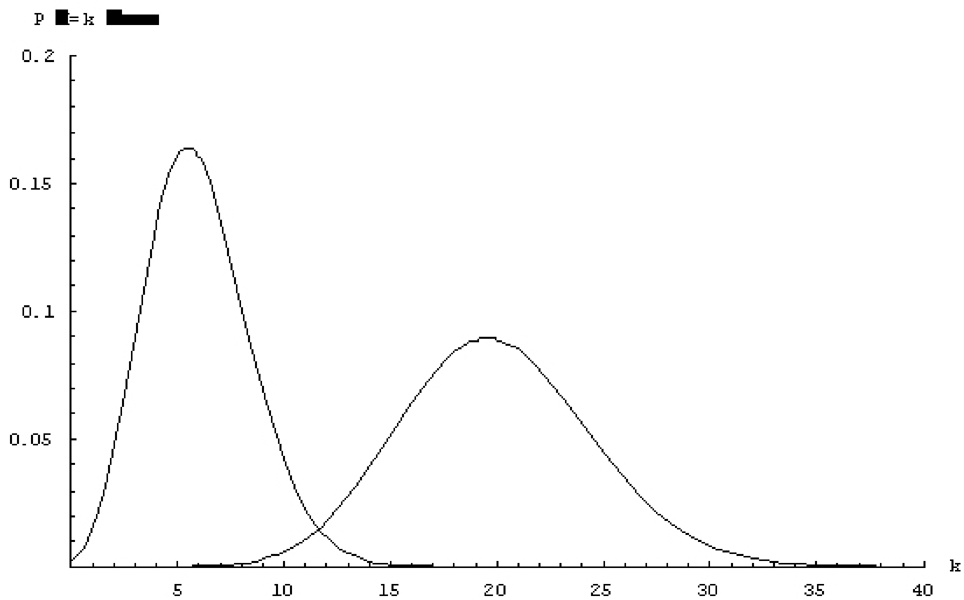
This will significantly improve the customer satisfaction. Certainly this will also bring more customers to the shop and make fewer customers leave instead of waiting.

M1: POISSON PROCESSES

A stochastic process *X(k)* is a (time-homogeneous, one-dimensional) Poisson process if the probability of the number of events in the interval [*0,k*] is given by

$$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda} \tag{1.1}$$

The following graph shows the Poisson distribution for $\lambda=5$ and $\lambda=20$.



In our model the Poisson distribution defines the probability for serving k customers per hour ($\lambda=6$) and for k customers entering the shop ($\lambda=20$.).

M2: CLASSIFICATION PATTERN FOR WAITING QUEUE SYSTEMS

Based on D. G. Kendall (1951) waiting queue systems are described using the following notation:

A/B/s

with

- A: distribution of the arrival times
- B: distribution of the service times
- s: number of servers

Shortcuts for distributions

- M: Markov distribution (exponential distributed times)
- G: general distribution (times not specified)
- D: deterministic distribution (constant times)

M3: ARRIVAL RATE

The arrival rate λ is defined as

$$\lambda \equiv \lim_{t \to \infty} \overline{A}(t) \tag{3.1}$$

$$\text{with } \overline{A}(t) \equiv \frac{A(t)}{t} \tag{3.2}$$

$A(t)$ is the number of customers arriving in the system in the period [0, t].

$\overline{A}(t)$ is the average number of arrivals in the [0, t] and λ the long term mean.

M4: SERVICE RATE

The service rate μ is defined as

$$\mu \equiv \lim_{t \to \infty} \overline{D}(t) \tag{4.1}$$

$$\text{with } \overline{D}(t) \equiv \frac{D(t)}{t} \tag{4.2}$$

$D(t)$ is the number of customers served in [0, t]. $\overline{D}(t)$ is the average number of served customers in [0, t] and μ is the long term mean. μ is given for a server, not for a system (see M7).

M5: SERVICE TIME

The service time is defined as the average time for serving a customer.

$$\overline{t_B} \equiv \frac{1}{\mu} \tag{5.1}$$

M6: TRAFFIC DENSITY

The traffic density ρ, also known as load factor, describes the work load of a system.

$$\rho \equiv \frac{\lambda}{m \cdot \mu} \tag{6.1}$$

m: Number of servers in the system, in our model shop the number of desks

M7: SERVICE RATE OF THE SYSTEM

$$\mu_{System} \equiv m \cdot \mu \qquad (7.1)$$

M8: PROBABILITY THAT NO CUSTOMER IS IN THE SYSTEM

The probability *P*(*X*=0) is given by the balanced condition of a stable system. The derivation exceeds the scope of this document.

$$P(X=0) = \frac{1}{\left[\sum_{n=0}^{m-1} \frac{1}{n!} \cdot \left(\frac{\lambda}{\mu}\right)^{n}\right] + \frac{1}{m!} \cdot \left(\frac{\lambda}{\mu}\right)^{m} \cdot \frac{m\mu}{m\mu - \lambda}} \qquad (8.1)$$

M9: PROBABLITY THAT A CUSTOMER HAS TO WAIT

$P(X \geq m)$ defines the probability that more or equal customers (X) than servers (m) are in the system. This means all servers are occupied.

$$P(X \geq m) = \frac{1}{m!} \cdot \left(\frac{\lambda}{\mu}\right)^{m} \cdot \frac{m\mu}{m\mu - \lambda} \cdot P(X=0) \qquad (9.1)$$

M10: AVERAGE LENGTH OF THE WAITING QUEUE

$$L_q = \frac{\lambda}{m\mu - \lambda} \cdot P(X \geq m) \qquad (10.1)$$

M11: AVERAGE WAITING TIME IN THE QUEUE

$$W_q = \frac{L_q}{\lambda} \qquad (11.1)$$

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